An $O(\ln n / \ln\ln n)$-approximation Algorithm for the Asymmetric Traveling Salesman Problem and its Prerequisites

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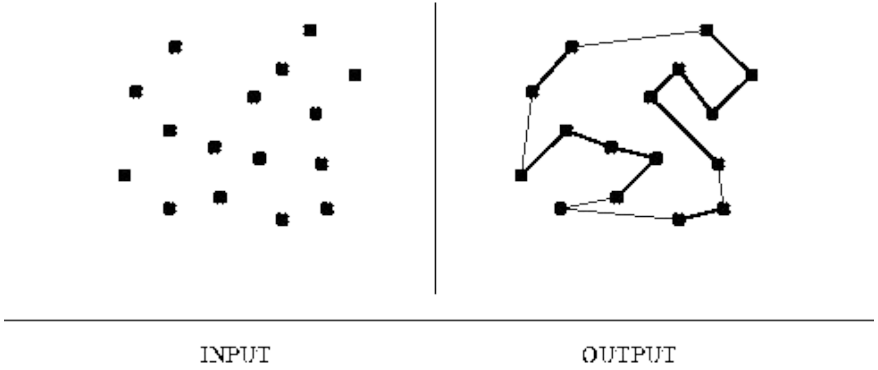
Asymmetric Traveling Salesman Problem (ATSP)

- As a Practical Problem



- Formal Definition

Graphs
MAC0320

Given a complete digraph $D = (V, A)$ *and a cost* $c \geq 0$ *on the arcs, find a minimum cost cycle that traverses each vertex of* D *exactly once.*

How to attack it?

- TSP and ATSP are NP-hard

  Algs
  MAC0338

- Moreover, they cannot be approximated unless P = NP
- However, their *metric* versions can (mTSP and mATSP)
  - For each u, v, w in V, we impose $c_{uw} \leq c_{uv} + c_{vw}$
- Thus, I will show the approximation algorithm developed by **(Asadpour et al., 2010)** for the mATSP

  Approx
  MAC0450

Approximation Algorithms

- Optimization Problem of Minimization with OPT
    - An algorithm is an α-approximation algorithm if it returns a candidate whose cost is at most $\alpha\,\mathrm{OPT}$ where $\alpha \geq 1$
- Paradigm: Lose optimality, Gain efficiency with guaranteed quality

Christofides Algorithm and Asadpour et al. Algorithm

Input: Graph **G,** metric cost $c \geq 0$ (mTSP)

1. Find a MST **T** in **G**
2. Transform **T** into an Eulerian graph **G'** with a min-cost perfect matching involving vertices of **T** of odd degree
3. Find a closed walk **W** that traverses each edge of **G'** once
4. Shortcut **W**

Input: Digraph **D,** metric cost $c \geq 0$ (mATSP)

1. Find opt-sol **x*** to Held-Karp relaxation of mATSP
2. Find **T*** that is $(\alpha, 2)$-thin tree "wrt" **x*** with high probability
3. Transform **T*** into an Eulerian digraph **D'** with a min-cost integer circulation of cost at most $(2\alpha + 2)\mathrm{OPT}_{\mathrm{HK}}$
4. Find a closed eulerian trail **W** in **D'**
5. Shortcut **W**

Find opt-sol x* to Held-Karp relaxation of mATSP

- Formulate mATSP as an optimization problem involving 0-1 variables and **exponentially many constraints**
- Allow fractional values in [0, 1] and obtain a **linear optimization program** called Held-Karp relaxation of mATSP

  LinProg
  MAC0315
- Equivalence optimization and separation problems helps to solve HK relaxation in polynomial-time
  - Ellipsoid Method

    LinAlg
    MAT0122
- Reduction to Max-flow Min-Cut and Flow Algorithms

  CombOpt
  MAC0325

Find T* that is a $(\alpha, 2)$-thin tree "wrt" x* with high probability

- We want a 0-1 vector that represents a spanning tree with a certain structure (thin-tree)
- However, we have a fractional vector **x***
- Rounding
  - Randomized Swap Rounding (RSR) by **(Chekuri et al., 2009)**
- Spanning tree sampled from RSR is α-thin with high probability.
  - Chernoff Bounds (Concentration Bound) Prob MAE0121
  - Result on counting β-minimum cuts due to **(Karger, 1993)**

Conclusion

- Important breakthrough for an important problem
- Wide and interesting connection of areas
    - Linear Algebra, Graph Theory, Probability Theory, Linear Programming, Combinatorial Optimization, Approximation Algorithms, Analysis of Algorithms

Bibliography

Asadpour, A., M. Goemans, A. Madry, S. O. Gharan, and A. Saberi. 2010. “An O(logn/loglogn)-approximation Algorithm for the Asymmetric Traveling Salesman Problem.” *In Proceedings of the Twenty-First ACM-SIAM Symposium on Discrete Algorithms*.

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Thank you!